

Exercise 54

Let $f(x) = \llbracket \cos x \rrbracket$, $-\pi \leq x \leq \pi$.

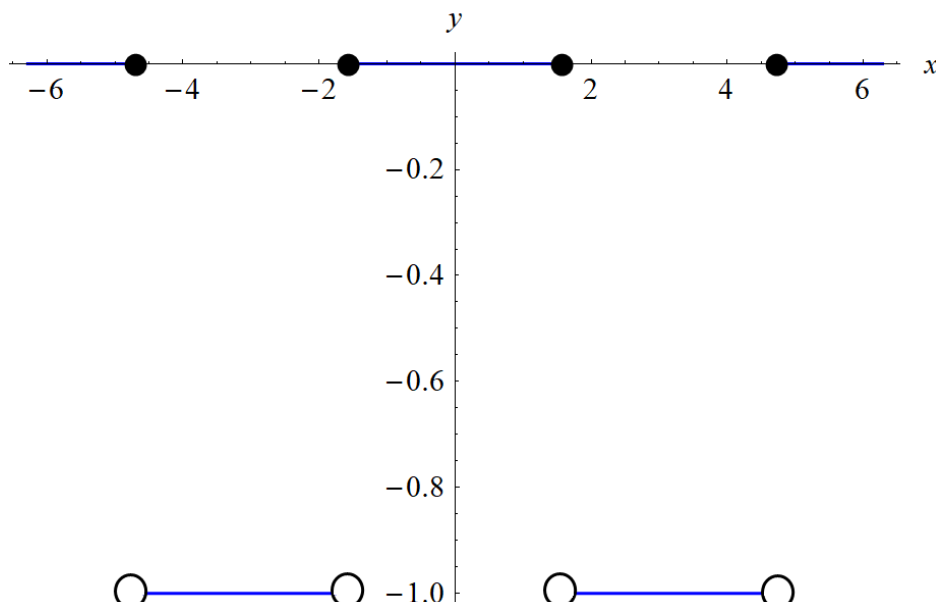
- (a) Sketch the graph of f .
- (b) Evaluate each limit, if it exists.

$$\begin{array}{ll} \text{(i)} \quad \lim_{x \rightarrow 0} f(x) & \text{(ii)} \quad \lim_{x \rightarrow (\pi/2)^-} f(x) \\ \text{(iii)} \quad \lim_{x \rightarrow (\pi/2)^+} f(x) & \text{(iv)} \quad \lim_{x \rightarrow \pi/2} f(x) \end{array}$$

- (c) For what values of a does $\lim_{x \rightarrow a} f(x)$ exist?

Solution

Below is a graph of $y = f(x)$.



Use it to evaluate the limits in question.

$$\lim_{x \rightarrow 0} f(x) = 0$$

$$\lim_{x \rightarrow (\pi/2)^-} f(x) = 0$$

$$\lim_{x \rightarrow (\pi/2)^+} f(x) = -1$$

$$\lim_{x \rightarrow \pi/2} f(x) \text{ does not exist because } \lim_{x \rightarrow (\pi/2)^-} f(x) \neq \lim_{x \rightarrow (\pi/2)^+} f(x)$$

$\lim_{x \rightarrow a} f(x)$ exists whenever $a \neq (\pi/2)(2n+1)$, where n is an integer.